Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

On Mathematical Concepts of the Material World. A. N. WHITEHEAD, Sc.D., F.R.S. *Philosophical Transactions of the Royal Society of London,* Series A, Vol. 205, 1906. Pp. 465–525.

This paper represents an attempt to develop concepts of a material world in a manner similar to that in which concepts of mathematical systems are now commonly developed. By a mathematical concept of the material world is meant a defined system of relational types such that if the proper entities are chosen as the terms of some specific system of relations conforming to these types, the system will represent known truths of the physical world. At least this is true of the five concepts which are here developed, although the abstract statement of the problem merely requires that a material world be a set of entities forming the field of a polyadic relation R and having the axioms of geometry expressive of certain properties of the field of R. The uninterpretable concepts are, however, avoided just as the students of pure mathematics are accustomed to leave undeveloped those concepts which admit of no useful specification.

Accepting Mr. Whitehead's further limitation to worlds depicted by three-dimensional Euclidean geometry, we recognize the classical concept of the material world as involving relations between three sorts of entities, points of space, particles of matter and instants of time, the first two of which are grouped together, without metaphysical implication, under the name of 'objective reals.' The class of 'objective reals' is, therefore, dualistic, and the first attempt at simplification naturally aims at doing away with this dualism.

The second concept, which is derived from Mr. Russell, starts from the fact that it is the essential function of matter to correlate some points of space with all instants of time and therefore substitutes for the class of particles a class of relations having as their domain a three-dimensional continuum, space, and for their converse domain a one-dimensional continuum, time. Mr. Whitehead questions the utility of this concept on the ground that relations can not be perceived, but in doing so he has departed from the standpoint of pure mathematics, from which it could be replied that it is the specifications of the concept and not the concept which come in question for perception, and even if relations should be unperceivable, so long as we can specify the correlating relations as particles of matter the concept is just as good as any other. It is a question, however, whether this concept is practically a simplification.

Another way of avoiding the dualism of 'objective reals' is to make use of the Leibnizian notion of relative space, and let points of space move. All geometrical theorems must then be correlated with an instant of time when referred to particular points or classes of points, and some sort of 'kinetic axes' must be introduced to give meaning to the assertion that a point is at any time in the position occupied by another point at some other time. Points are differentiated merely by their motions, and if by persistence of volume we mean persistence of the same points in the same relative motions, we have a conception close to that of Lord

Kelvin's vortex rings. We may, however, interpret persistence of volume merely as persistence of the relative motion of a complex of points the individuals of which are constantly changing.

The last two concepts are developed from a linear entity somewhat analogous to directions and lines of force, called by Mr. Whitehead a 'linear real.' We get in this way quite complex concepts, but they are very suggestive of physical ideas. For example, the fifth concept involves five different kinds of points, some of which may be looked upon as positive and negative electrons according to certain characteristics of the 'linear reals' which they contain. It is remarked, however, that we still lack some simple hypothesis concerning the motion of the 'objective reals' to correlate them with electric points and electrons. "From such an hypothesis the whole electromagnetic and gravitational laws *might* [italics mine] follow with the utmost simplicity." The attractive aspect of the concept is that by it the properties of space and of objects in space are derivable from one sort of entity.

Greatly as the production of these concepts testifies to Mr. Whitehead's ingenuity, they are too remote from ordinary thought and too incomplete in critical points such as that quoted above to make one feel that the labor of either author or reader is quite justified, but there are two preliminary studies to the linear concepts which take up a large portion of the paper and are immensely worth while to the student of the logic of mathematics. They are the 'Theory of Interpoints' and the 'Theory of Dimensions.' If a linear element is taken as fundamental we must be able to define a point, 'the intersection of lines at a point,' without the circularity here manifest and without the distinction of the descriptive and the projective point made use of in ordinary geometry. The above-named theories do this very prettily in two distinct fashions.

The 'Theory of Interpoints' (intersection points) starts with a class of 'objective reals,' perfectly simple, indivisible entities, and a relation $R;(abcdt)$ which we are told may be read 'the objective real a intersects the objective reals b, c, d in the order bcd at the instant t.' But to avoid the suggested connotation of 'intersects' we may equally well read $R;(abcdt)$ 'the banker a has business dealings with the bankers b, c, d of importance varying in the bcd order at the time t.' The advantage of the otherwise unbearable symbolism of the text is that all accidental connotations are thus readily avoided. We can then say that a banker y has a position in the relation R similar to that of a banker x when a has business dealings of the same importance with both x and y at the same time. We can further define a business set as the class of bankers having similar positions with respect to a and including a. All the business sets of which a is the ruling member constitute a series of sets. Now a business set so defined is an accurate specification of Mr. Whitehead's 'interpoint,' and the series of business sets represents the 'punctual line' which is associated with the 'linear real' a. What is meant by the meeting of two lines at a point is, therefore, defined without circularity, and we have a beautiful illustration of the truth that pure mathematics concerns itself with types of relations only, abstracting from all possible specifications of the relations and utterly heedless of the entities which may stand as their terms.

The 'Theory of Dimensions' is especially interesting from the fact that it does not involve ordinal properties in the accomplishment of its end. We begin by defining a ϕ-class, *i. e.,* a class such that if u is a ϕ-class the proposition $\phi ! u$ is true, and then by definitions of ϕ-regions and subregions we come upon certain properties of some ϕ-classes such that we can say that any class having those properties is a ϕ-point. From this point on it is a straight road to develop a complete geometry and finally arrive at a concept of a material world, but it is in these subsidiary theories and not in the concepts themselves that we believe the highest value of the paper lies.

HAROLD CHAPMAN BROWN.

COLUMBIA UNIVERSITY.

Aktionsströme vom Gehörorgan der Fische bei Schallreizung. H. PIPER. *Zentralblatt für Physiologie,* Bd. XX., Nr. 9.

Die akustischen Funktionen des inneren Ohres und seiner Teile. H. PIPER. *Medizinische Klinik,* 1906, Nr. 41.

Although the ears of fishes possess no cochlea, Zenneck and G. H. Parker have shown that these vertebrates respond, nevertheless, to sound stimuli. This response must, then, be mediated by some or all of the sense-organs in the ampullæ, utriculus and sacculus of the ear, or else by some organ outside of the ear. Investigators who hold the Mach-Breuer theory that, in the higher vertebrates at least, the cochlea alone is the organ of hearing have suggested that the hearing of fishes is probably mediated by sense-organs located in the skin. But Parker has shown, by severing the sensory nerves leading from the lateral line and other parts of the skin, that in some species at least the response to sound vibrations is mediated by the cochlealess ear. Piper has now studied this response, in the pike (*Esox lucius*) and eel, by means of action-currents similar to those demonstrated for the retina by Holmgren. Earlier work with these currents has shown that in general any active organ or tissue is electronegative toward other near-lying and non-active organs or tissues; and if when both are at rest the former shows a positive difference of potential toward the latter, this positivity (rest-current) is decreased when the former is excited. "Such fluctuations of potential, or action-currents, are a sure indication of the presence of functional changes in the organism."

Piper applied one electrode to the large otolith of the sacculus (or to that of the utriculus) and the other to the exposed section of the acoustic nerve, or to a near-lying part of the inner surface of the skull. The rest-current shows the otolith positive while the nerve section is fresh, but this current is gradually reversed as the exposed end begins to die (*absterben*). The sounds were produced under water by two whistles, of 100 and 260 vs. The action-currents always showed an increased negativity of the otoliths, and they subsided as soon as the sound ceased. "The deflection of the needle is greater when the sound stimulus lasts